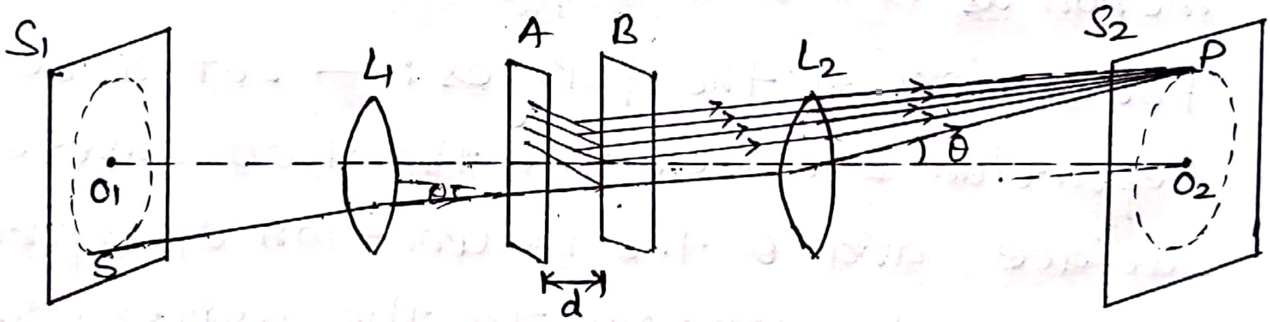
prof. vidya Sagar
Department of physics
BSc (part-II) Hons

Constriction and working of a Fabry-perot interferometer. Measurement of wavelength of a monochromatic light.

Fabry-perot interferometer:- It is a high resolving power instrument which makes use of the fringes of constant inclination produced by the transmitted light after multiple reflections in an air film between two parallel and highly reflecting glass plates.



It consists of two optically plane glass plate A and B with their inner surface silvered, and placed accurately parallel to each other. Screws are provided to secure parallelism if disturbed. The plate themselves are slightly prismatic to avoid interference among the rays reflected at the outer unsilvered surface.

S_1 is a board source of monochromatic light and L_1 a convex lens which makes the rays parallel. An incident rays suffers a large number of internal reflection successively at the two silvered surface as shown. At each reflection a small fractional part of the light is transmitted also. Thus each incident ray produces a group of ~~cho~~ coherent and parallel transmitted ray with a constant path difference between any two successive rays. A second convex lens L_2 brings these rays together at a point P in its focal plane where they interfere. Hence the rays from all points of the source produce an interference pattern on a screen S_2 placed in the focal plane of L_2. The phenomenon is called multiple beam in the interference

Formation of the fringes :— Let d be the sepration between the the two silvered surface, and θ the inclination of a particular ray with the normal to the plates. Then the path difference any two successive transmitted rays corresponding to the incident ray is $2d\cos\theta$. The condition for these rays to produce maximum intensity is given by

$$2d\cos\theta = n\lambda$$

Where n is an integer, called the order of interference, and λ the wavelength of light, The focus of points in the source which give rays of a constant inclination θ is a circle. Hence with an extended source, the interference pattern consists of a system of bright concentric rings on a dark background each ring corresponding to a particular value of θ.

Intensity Distribution:- Let T and R be the fractions of the incident light intensity which are respectively transmitted and reflected at each silvered surface. Then the fractional ~~transmitted~~ transmitted and reflected amplitudes will be $\sqrt{T}$ and $\sqrt{R}$. The amplitudes of the successive rays ~~transmitted~~ through the pair of plates will be.

$aT, aTR, aTR^2, ---$ where a is ~~a~~ the ~~ampli~~ incident amplitude.

The phase difference δ between any two rays reaching a point on the screen is given by

$$\delta = \frac{2\pi}{\lambda} \times \text{path difference}$$

$$\delta = \frac{2\pi}{\lambda}(2d\cos\theta) \quad \text{—— (1)}$$

If the incident wave is represented by $y = ae^{i\omega t}$, then the waves reaching a point on the screen will be

$$y_1 = aTe^{i\omega t}$$

$$y_2 = aTRe^{i(\omega t-\delta)}$$

$$y_3 = aTR^2e^{i(\omega t-2\delta)}$$ and so on

by the principal of superposition, the resultant amplitude is given by

$$A = aT + aTRe^{-i\delta} + aTR^2e^{-2i\delta} + aTR^3e^{-3i\delta} + \text{------}$$

$$= aT\left(1 + Re^{-i\delta} + R^2e^{-2i\delta} + R^3e^{-3i\delta} + \text{-----}\right)$$

$$A = aT\left(\frac{1}{1-Re^{-i\delta}}\right)$$

The complex conjugate of A is therefore

$$A^* = aT\left(\frac{1}{1-Re^{+i\delta}}\right)$$

Hence the resultant intensity I is given by

$$I = AA^*$$

$$= \frac{a^2T^2}{(1-Re^{-i\delta})(1-Re^{+i\delta})}$$

$$= \frac{a^2T^2}{1+R^2-2R(e^{i\delta}+e^{-i\delta})}$$

$$= \frac{a^2T^2}{1+R^2-2R\cos\delta}$$

$$= \frac{a^2T^2}{(1-R)^2+2R(1-\cos\delta)}$$

$$= \frac{a^2T^2}{(1-R)^2 + 4R\sin^2\frac{\delta}{2}}$$

$$= \frac{a^2T^2}{(1-R)^2}\left[\frac{1}{1+\frac{4R}{(1-R)^2}\cdot\sin^2\frac{\delta}{2}}\right] \quad \text{——— (2)}$$

Then intensity will be maximum when

$\sin^2(\delta/2) = 0$, i.e. $\delta = 2n\pi$

where $n = 0,1,2,3,---$

Thus $I_{max} = \dfrac{a^2T^2}{(1-R)^2}$ ——— (3)

Similarly the intensity will be a minimum when

$\sin^2(\delta/2) = 1$, i.e. $\delta = (2n+1)\pi$

where $n = 0,1,2,-----$

$$\therefore I_{min} = \frac{a^2T^2}{(1+R)^2}\left[\frac{1}{1+\frac{4R}{(1-R)^2}}\right]$$

or, $I_{min} = \dfrac{a^2T^2}{(1+R)^2}$ ——— (4)

$\therefore$ from (2) we have

$$I = \frac{I_{max}}{1+\frac{4R\sin^2(\delta/2)}{(1-R)^2}} \quad \text{——— (5)}$$

This is required equation for the intensity for the F.P. interferometer.

Measurement of wavelength: — In a febry perot interferometer the condition for maximum is

$$2d \cos\theta = n\lambda \quad \text{—— (6)}$$

Where n is the order of interference and λ is the wavelength of light used. At the Centre of the fringe System $\theta = 0$, i.e. $\cos\theta = 1$

$$\therefore 2d = n_0\lambda \quad \text{—— (7)}$$

Where n_0 is the order of interference at the centre. Let us suppose that corresponding to a separation d_1 between the plates there is a bright fringe at the Centre, and corresponding to another Separation d_2 there is again a bright fringe at the centre, then from equation (7) we have

$$2d_1 = n_1\lambda \text{ and } 2d_2 = n_2\lambda$$

$$\therefore 2(d_2 - d_1) = (n_2 - n_1)\lambda = N\lambda$$

Where $N = (n_2 - n_1)$ is the number of fringes that cross the Centre of the field of view when the plate separation is changed from d_1 to d_2

$$\therefore \boxed{\lambda = \frac{2(d_2 - d_1)}{N}} \quad \text{—— (8)}$$

Thus knowing d_1, d_2 and N then λ can be Calculated.